PRINCIPLES of COUNTING and QUANTITY

TAPINTOTEENMINDS

In This Guide, We'll Learn:

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Success in mathematics begins with the development of a sense of number through **counting and quantity**. It may seem that children learn to count in much the same way that they learn their alphabet - by simply repeating numbers by rote. While teaching our children to count from 1 to 10, 20 or even 100 is helpful, this skill alone is not the same as learning how to count. For children to build number flexibility in primary and middle school grades, developing a strong foundation in counting and quantity is very important.

When a student begins to struggle along their mathematics learning journey, the culprit can often be linked to gaps in learning from previous grades. Having worked with many students in remedial math classrooms, I often found that struggles can be linked to a lack of understanding in counting and quantity. This Principles of Counting and Quantity Cheat Sheet is designed to help parents and teachers better understand how to help their children with counting and quantity during the early years.

Note: Most of the images and buttons below are clickable, so you can check out these resources in real time.

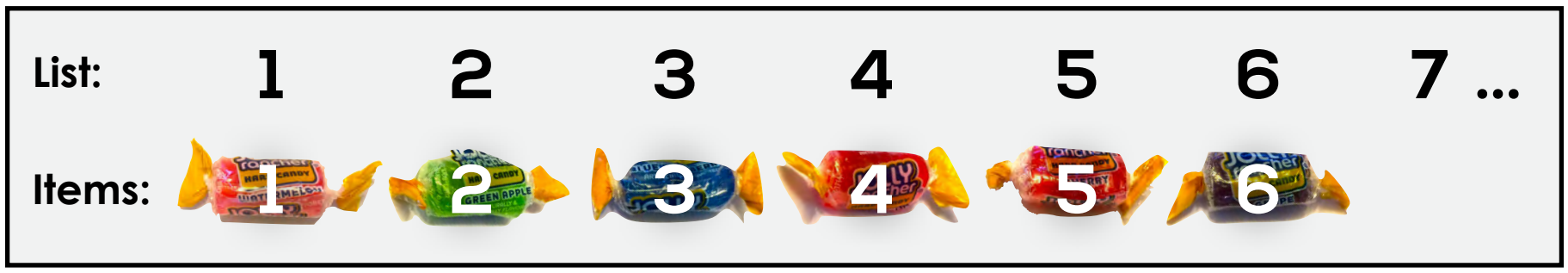
Principles of Counting and Quantity

Despite the fact that Rochel Gelman and Randy Gallistel introduced five principles of counting in 1978, this resource will introduce those five and an additional five that are very useful.

Click or visit: tapintoteenminds.com/counting-principles

Stable Order

The first principle of counting involves the student using a list of words to count in a repeatable order. This ordered or "stable" list of counting words must be at least as long as the number of items to be counted.



For example, if a student wants to count 20 items, their stable list of numbers must be to at least 20.

> *"The child uses a repeatable list of words that has more words than the number of items to be counted."*
>
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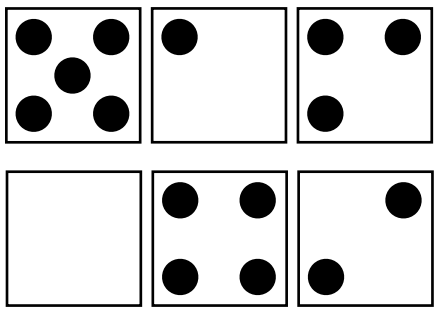
Thinking deeper about stable order, we might consider rote counting from 0, counting on from a number (i.e.: "start at 6 and count to 18") and counting backwards (i.e.: "count backwards from 15") skills that are related to stable order. Assisting students in acquiring these skills and flexibility to count on and count backwards will take time, but is helpful to building a deep understanding of counting and quantity.

Strategies that Support Student Learning

- Putting pictures of items in order from smallest to largest, based on quantity, and counting them forwards and backwards.
- Organizing objects in order, without numbers at first, then adding the symbols later.

Food Items



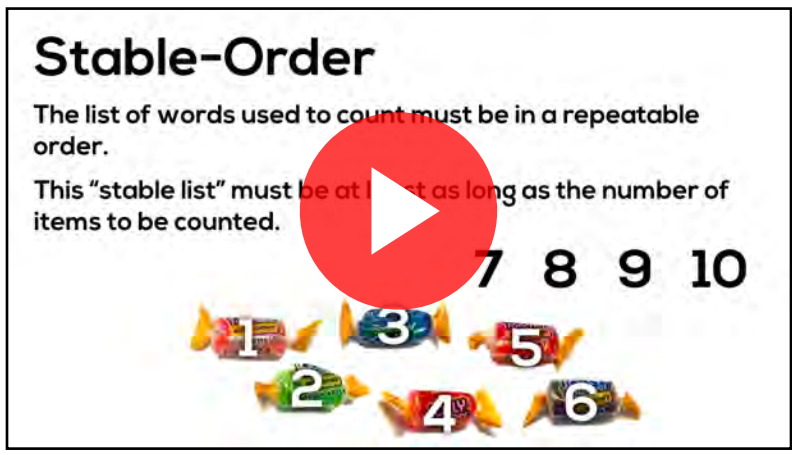
Dot Cards

If you observe:

- A child miscounting orally by rote or with objects…

Consider:

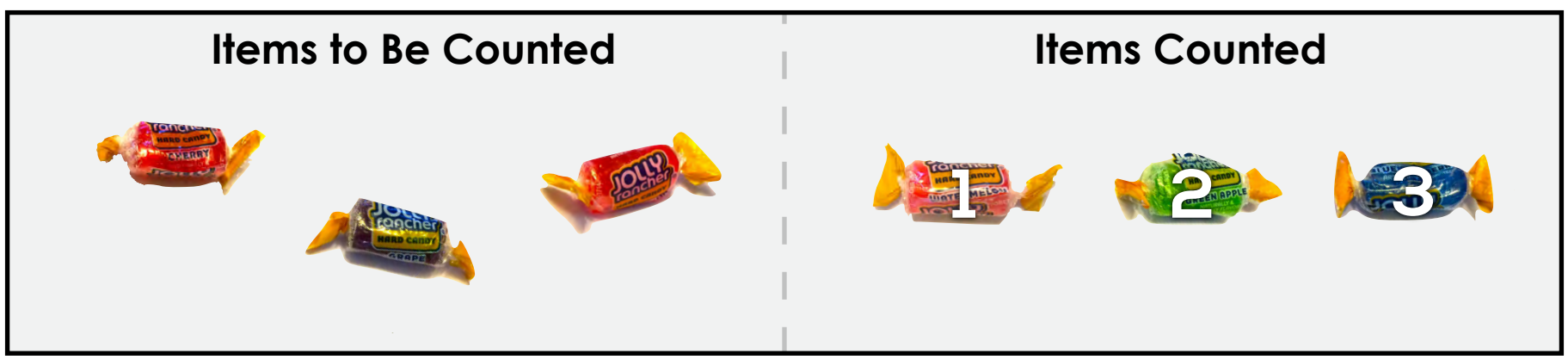
- Intentionally miscount and ask the child to tell you what number you missed.



Click or visit: mathisvisual.com/stable-order

One-to-One Correspondence

Understanding that each object in a group can be counted once and only once. It is useful in the early stages for children to actually tag or touch each item being counted and to move it out of the way as it is counted.



In order for students to understand and apply the one-to-one counting principle, they must be able to orally count by rote.

"The child counts each item in a group only once."

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We can promote the development of this skill by regularly counting items during play and everyday life. Encouraging students to show quantities on their fingers is also helpful.

Strategies that Support Student Learning

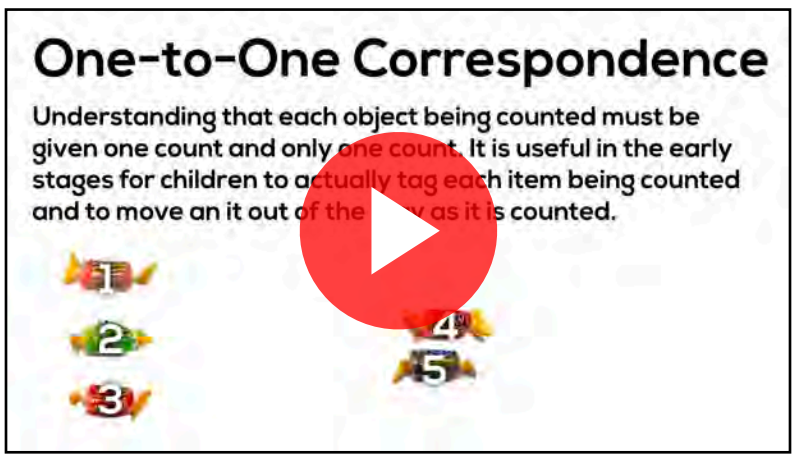
- Encouraging students to "tag" or move items out of the way while counting.
- Matching items with pictures. For example, using search and find books.
- Encouraging students to create a tally chart to count and track the quantity of food, toys, sounds (i.e.: taps on a drum), letters in a word or words in a sentence.

If you observe:

- A child playing in the kitchenette area preparing food for stuffed animals…

Consider:

- Asking how many items of food they are preparing or how many people they are cooking for.



Click above or visit:
mathisvisual.com/one-to-one-correspondence

Cardinality

Understanding that the last number used to count a group of objects represents how many are in the group.

A child who recounts when asked how many candies are in the set that they just counted, may not have an understanding of the cardinality principle.

If you are struggling to assess whether a student firmly grasps the cardinal principle, consider asking the student to count a group of items and then ask them to put the same quantity into a bag. If they must recount, they may not have a firm understanding of cardinality.

Strategies that Support Student Learning

- ☐ Encouraging students to show you a group of items to match a specific number.
- ☐ Ask students to count a group of items in a set. Then, explicitly ask them to show you how many objects in that group represent that amount.

If you observe:

- ☐ A child building a tower out of lego…

Consider:

- ☐ Asking if they can use the same amount of lego to create a path.

"The child understands that the last number used to count a group represents the magnitude of the entire group, not the last item."

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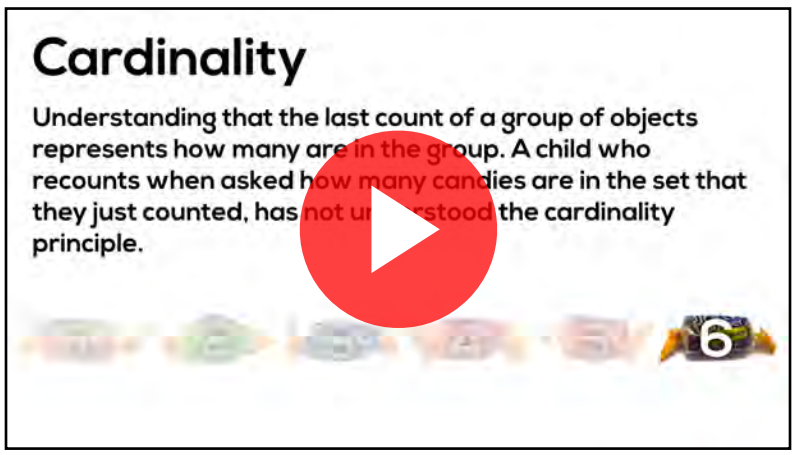

Click above or visit:
mathisvisual.com/cardinality

Conservation

Understanding that the count for a set group of objects stays the same no matter whether they are spread out or close together.

If a student counts a group of items that are close together and then needs to recount after you spread them out, they may not have developed an understanding of the principle of conservation.

"The child understands that whether you count a group of items that are close together or far apart, the count is the same."

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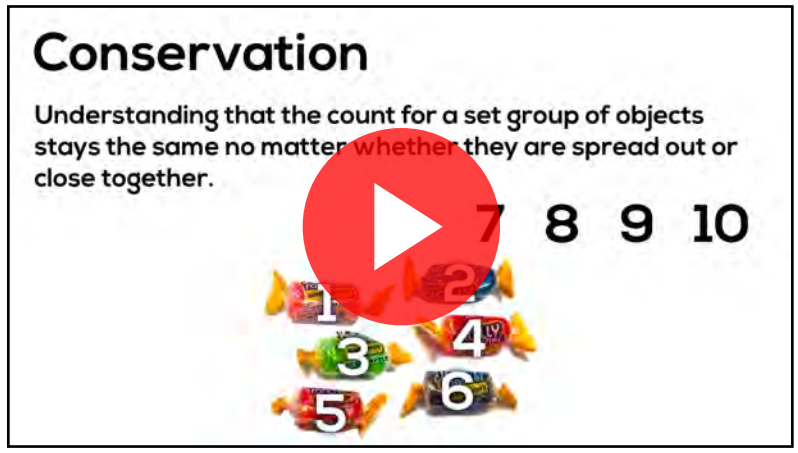

Click above or visit: mathisvisual.com/conservation

Counting and Quantity Tip: Avoid Rushing to Symbols

It is all too common that we rush towards symbols in mathematics and counting is no different. Help children develop a firm grasp of the quantity associated with each number concretely before we formally introduce the symbolic form of number.

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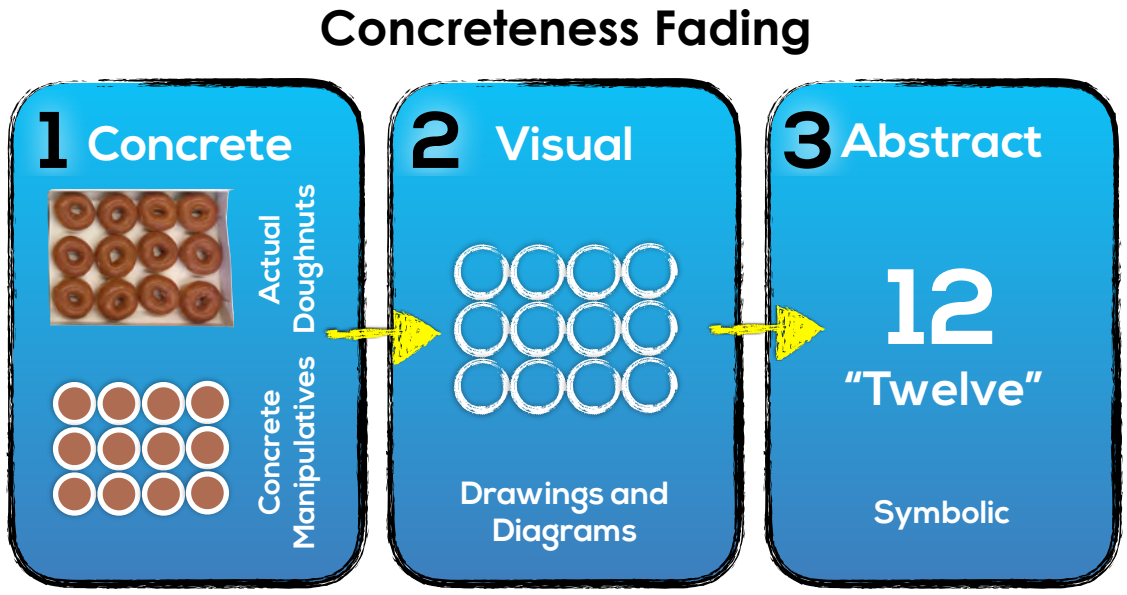

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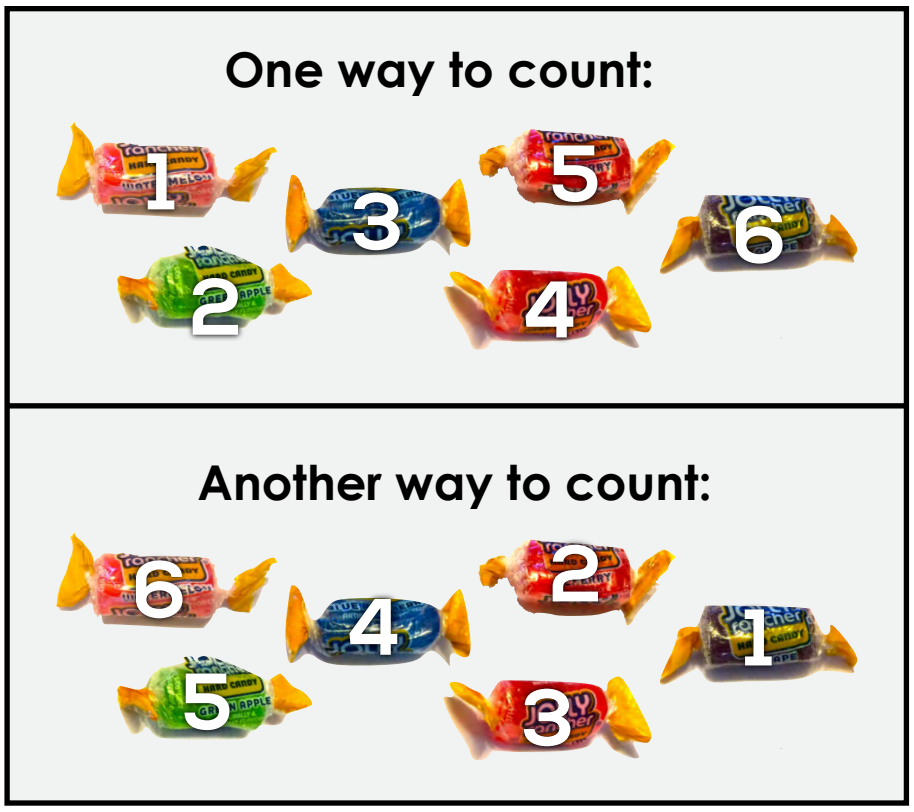
Order Irrelevance

The order in which items are counted is irrelevant.

Students have an understanding of order irrelevance when they are able to count a group of items starting from different places. For example, counting from the left-most item to the right-most and visa versa.



> *"The child is able to count a group of items starting from different places."*
>
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While the order irrelevance principle may seem obvious to adults, many students hold the misconception that the order you count objects ***does*** matter until as late as grade 4. Explicitly teaching this principle is important. It should also be noted that just because a child is strong in this principle, they may still be weak in other counting principles.

Strategies that Support Student Learning

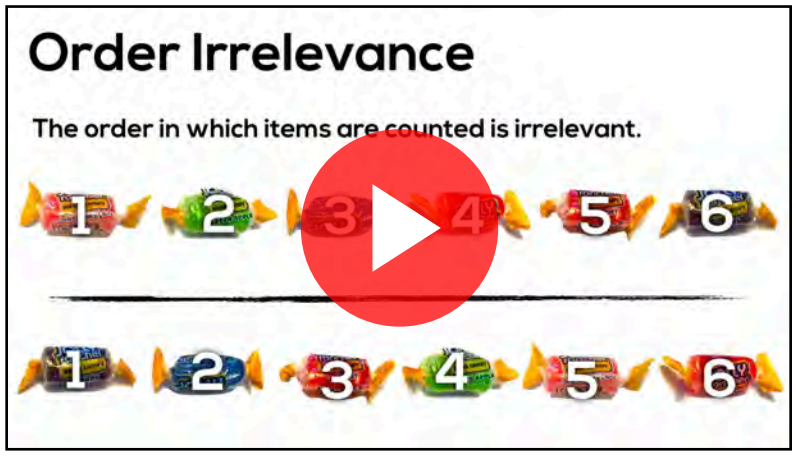
- ☐ Counting sets of items from **left-to-right**, **right-to-left**, **top-to-bottom** and **bottom-to-top**.
- ☐ Counting sets of unique items (different colour, shape, etc.) in a variety of orders.

If you observe:

- ☐ A child counting a set of toy cars…

Consider:

- ☐ Asking if they can predict how many cars there would be if they started counting from a different spot.



Click above or visit:
mathisvisual.com/order-irrelevance

Abstraction

Abstraction requires an understanding that we can count any collection of objects, whether tangible or not.

For example, the quantity of five large items is the same count as a quantity of five small items or a mixed group of five small and large things.

Another example may include a student being able to count linking cubes that represent some other set of objects like cars, dogs, or bikes.

"There are 5 candies."	"There are 5 coffees."
	

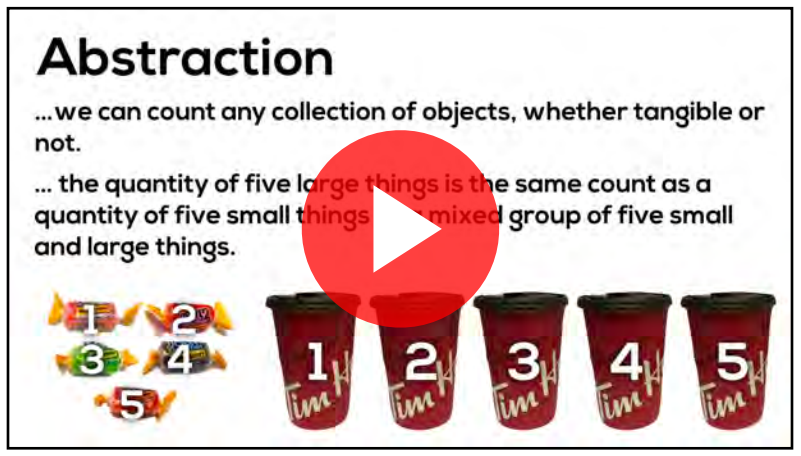
Children often consider groups of larger items to have more value than groups of smaller items. For example, a child may believe that the quantity of the 3 cars in the parking lot is larger than the 3 toy cars placed on the play mat

"It does not matter what you count, the way we count is always the same, whether tangible or not."

CLICK TO TWEET

Strategies that Support Student Learning

- ☐ Counting non-tangible quantities such as sounds, actions, words, questions or steps.
- ☐ Matching groups of different items with the same quantity.



Click above or visit: mathisvisual.com/abstraction

If you observe:

- ☐ A child playing with toys of different sizes…

Consider:

- ☐ Taking a group of 2 larger items and a group of 3 smaller items and asking which has more.

Subitizing

In general, subitizing is the ability to "see" or visualize a small amount of objects and know how many there are without counting. While this idea may seem simple on the surface, subitizing is actually quite complex. If we dig deeper, we can see that there are two types of subitizing that could be going on in our mind when we are learning to count called **perceptual subitizing** and **conceptual subitizing**.

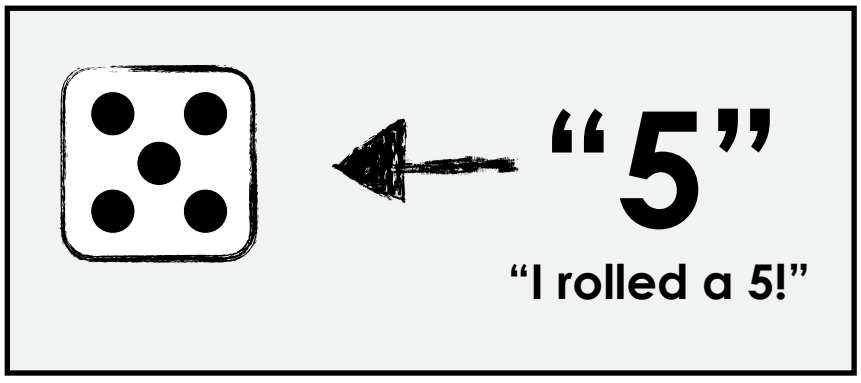
> *"The ability to 'see' or visualize small groups of objects and immediately know how many there are without counting."*
>
> CLICK TO TWEET

Perceptual Subitizing

Perceptual subitizing takes place when you are able to look at a group of objects and know how many objects there are without having to do any thinking. Often times, when we look at groups of 5 objects or less, we are subitizing perceptually.

Examples of perceptual subitizing could include:

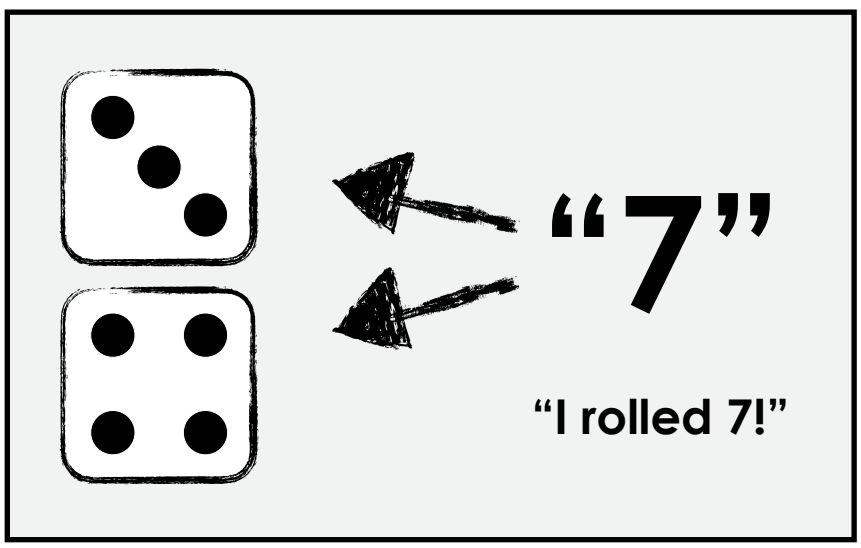
- knowing there are 3 candies on a table without counting the candies
- knowing you rolled 5 with a single die without counting the dots
- knowing there are 2 cars in your driveway without counting the cars



Conceptual Subitizing

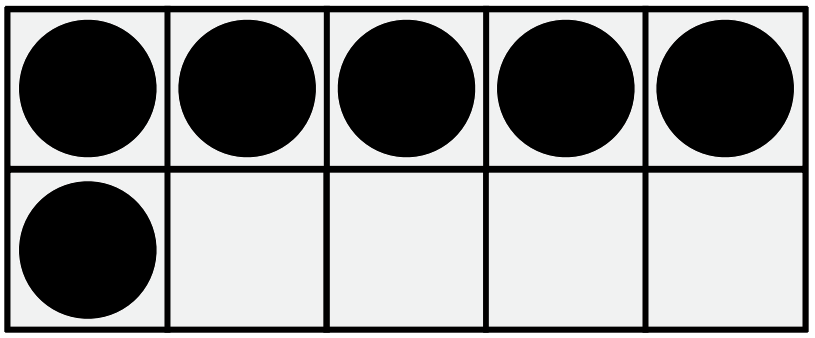
Conceptual subitizing takes place when you are still able to "see" how many objects are in group, but the number of objects is too large to subitize without decomposing into two or more smaller groups.

We often shift from perceptual subitizing to conceptual subitizing when the number of objects in a group is larger than 5.



You may find that you are able to perceptually subitize groups of more than 5 items when the items are organized in familiar ways. For example, most "know" they have rolled 6 on a single die because of the familiar arrangement of the dots. However, you may struggle to perceptually subitize those 6 dots if they were arranged in an unfamiliar way and resort to conceptually subitizing by breaking up the 6 dots into two groups of 3 in your mind without even realizing it!

While many may believe that using 5- and 10-frames in early mathematics is simply because 5's and 10's are very friendly numbers in our base 10 system, we can see that the organization of items in a 5-frame can be an important tool to help students shift from perceptual subitizing to conceptual subitizing.



10 Frame

Strategies that Support Student Learning

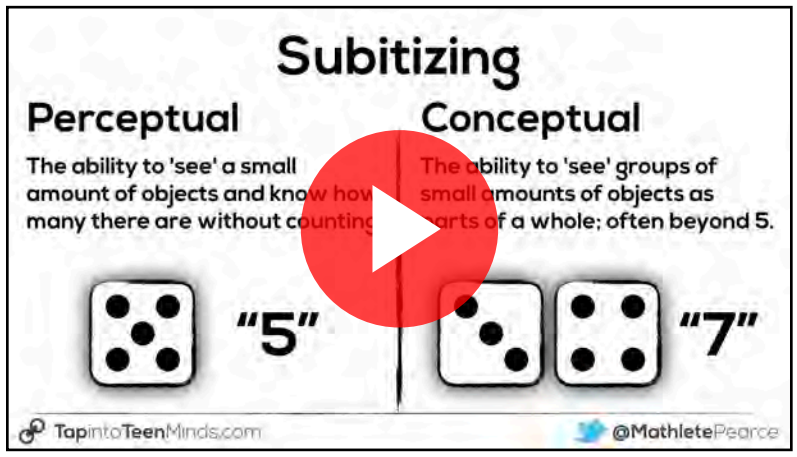
- ☐ Ask children to count how many objects are in a set that is out of reach or difficult to physically tag using one-to-one correspondence (i.e.: cars in the driveway, chairs at the table, etc.).
- ☐ Create dot cards using pieces of paper with small quantities of dots on each, arranged in different configurations and play matching games, war and other fun card games with them.

If you observe:

- ☐ A child playing with a small quantity of items…

Consider:

- ☐ Asking them how many items they are playing with.



Click above or visit: mathisvisual.com/subitize

Want More Subitizing Activities?

You can learn more about perceptual and conceptual subitizing and access useful activities by downloading the subitizing guide.

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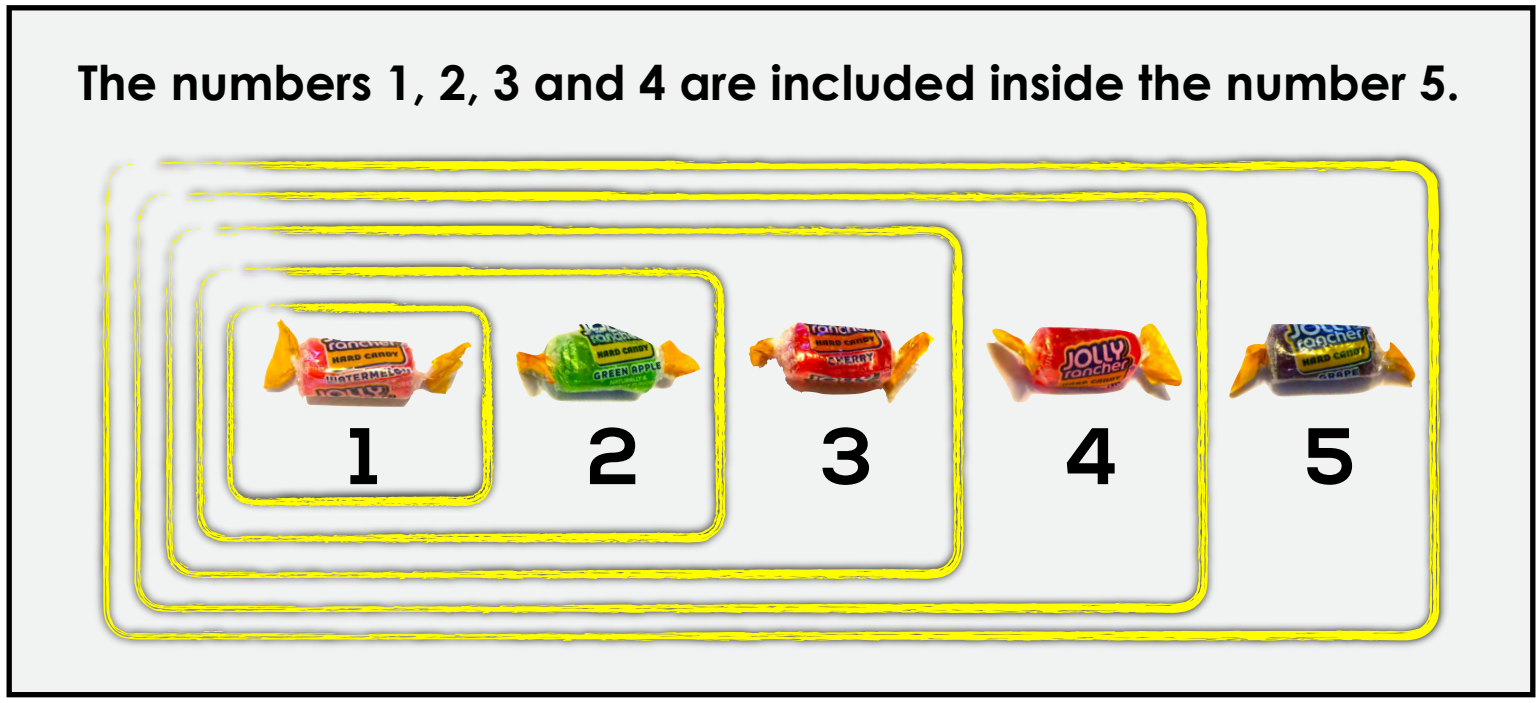
Or visit: kylep.ca/subitizeguide

Hierarchical Inclusion

Understanding that all numbers preceding a number can be or are systematically included in the value of another selected number.

For example, knowing that within a group of 5 items, there is also a group of 4 items within that group; 3 items within that group; 2 items… and so on.



Hierarchical inclusion is an important landmark that students must reach in order to fully understand cardinality and to begin composing numbers (i.e.: composing a set of 5 items by combining a set of 2 items with a set of 3 items) and decomposing numbers (i.e.: decomposing a set of 6 items by separating into a set of 4 items and a set of 2 items).

"The child understands that contained within a number are the other numbers before it."

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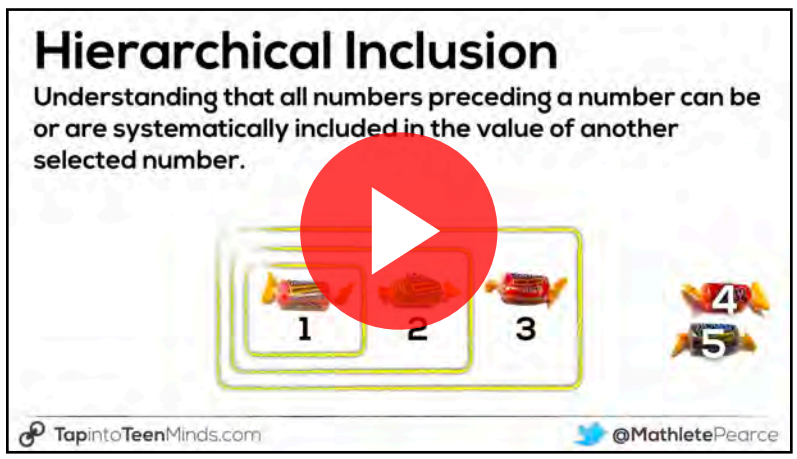
Strategies that Support Student Learning

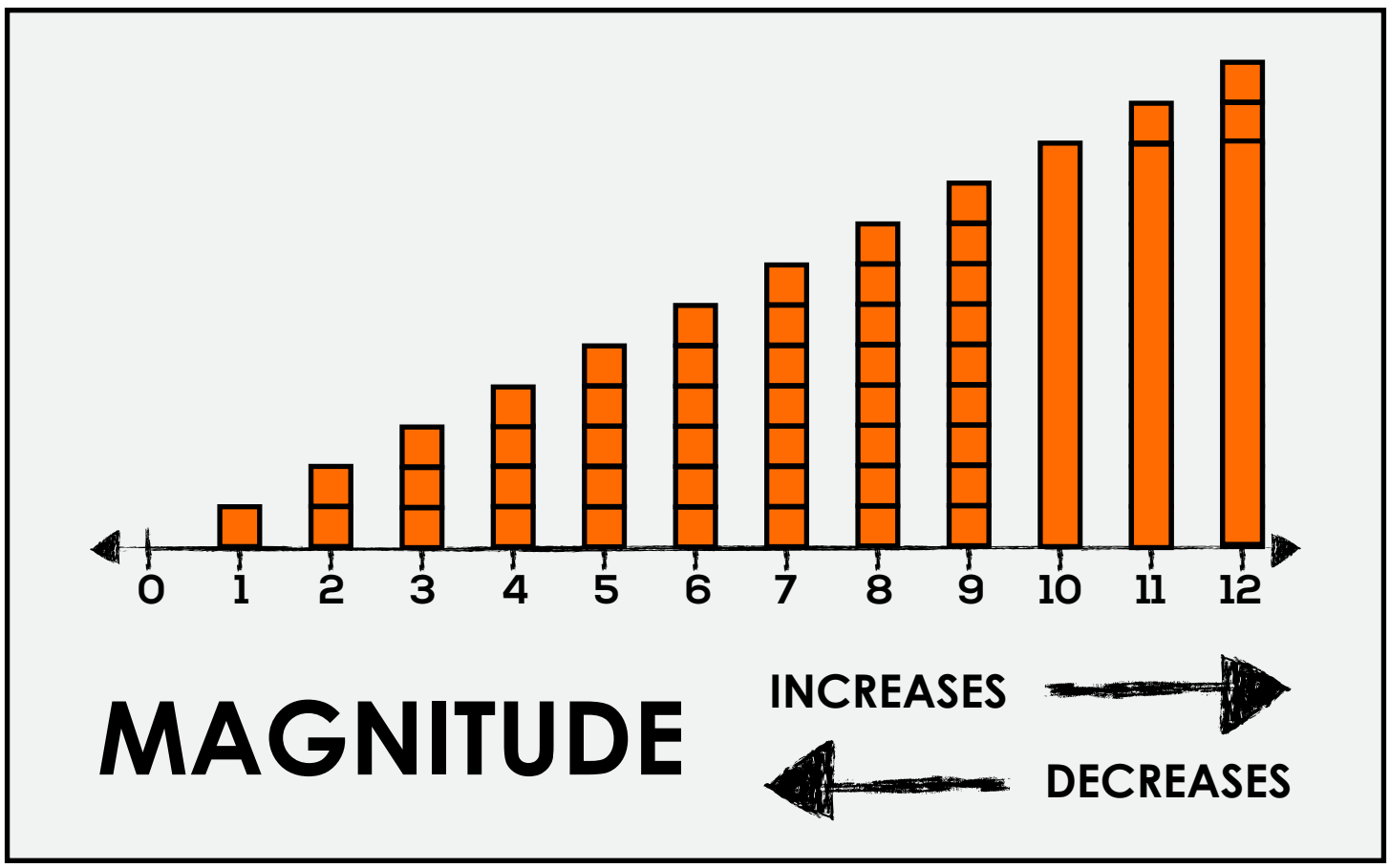
- Ask children to name the number that is "one larger" or "one smaller" than a number.
- Ask for a number that is "inside" the number 7.



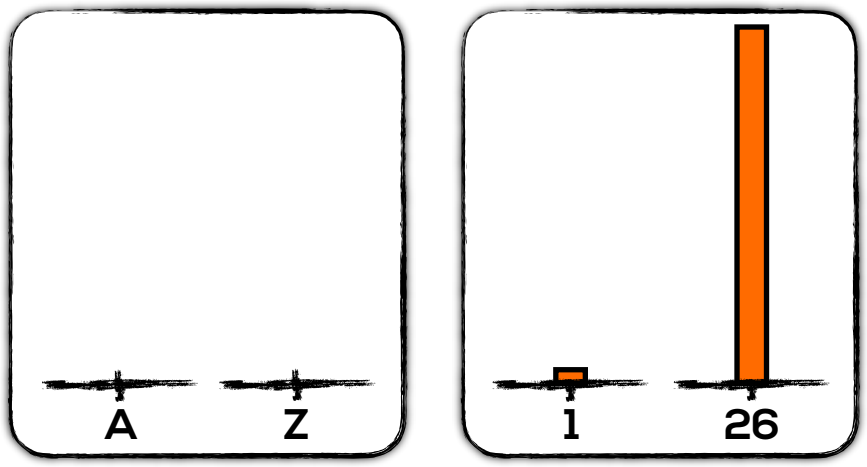
Click above or visit:
mathisvisual.com/hierarchical-inclusion

Movement is Magnitude

Understanding that as you move up the counting sequence (or forwards), the quantity increases by one and as you move down (or backwards), the quantity decreases by one or whatever quantity you are going up/down by.



Despite many believing that learning the alphabet is the same as learning how to count, our number system is much more complex than the alphabet. The letters in the alphabet are arranged in an order for no particular reason, whereas the stable list of numbers we use for measuring quantity through counting is ordered by magnitude.



"The child understands that as you count forward, the quantity increases; as you move backward, the quantity decreases."

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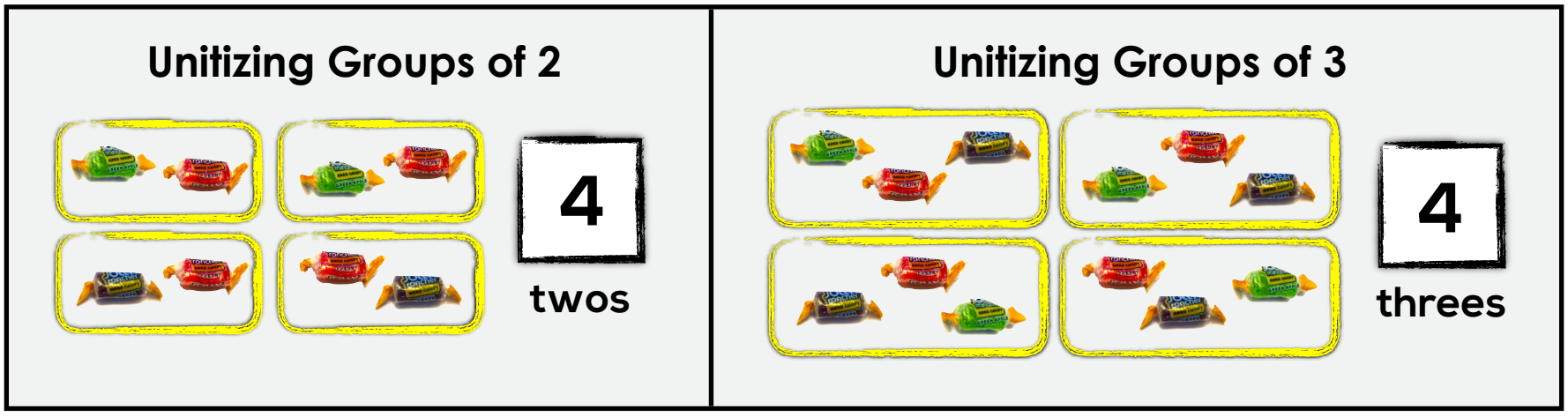
Click above or visit:
mathisvisual.com/movement-is-magnitude

Unitizing

Unitizing refers to the understanding that you can count a large group of items by decomposing the group into smaller, equal groups of items and then count those.

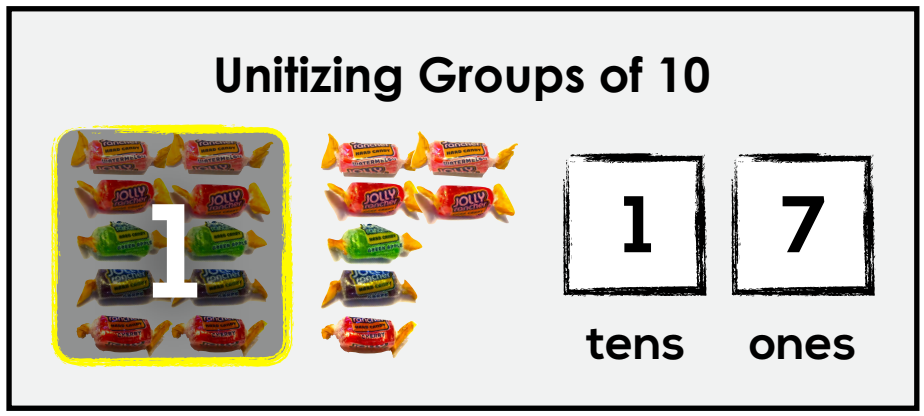
For example, if there is a large group of candies on a table, one might choose to create groups (or "units") of 2 (often doing this by perceptually subitizing these groups) and skip counting up by 2's. Some may choose to create "units" of 3 and skip count up by 3's.



We might consider connecting unitizing back to **1-to-1 correspondence** by thinking of unitizing groups of 2 as "**2-to-1 correspondence**" in the situation where we are counting 2 candies for every 1 group or unitizing groups of 3 as "**3-to-1 correspondence**" in the situation where we are counting 3 candies for every 1 group. I find by thinking of unitizing this way, it can begin revealing connections to multiplication and the underlying ratios that exist whenever we count any quantity.

Understanding Place Value Through Unitizing

Unitizing is also important for students to understand that objects are grouped into tens in our base-ten number system. For example, once a count exceeds 9, this is indicated by a 1 in the tens place of a number.



"The child is able to count equal groups of items rather than counting one at a time."

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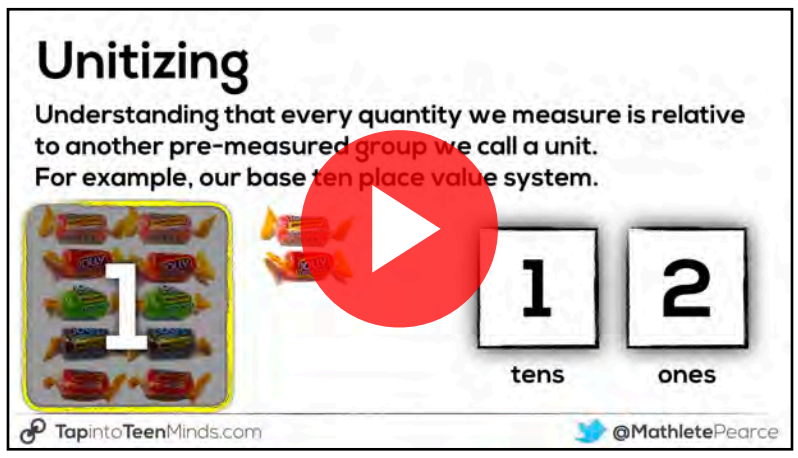


Click above or visit: mathisvisual.com/unitizing

Resources & References

Online Resources

Principles of Counting & Quantity
tapintoteenminds.com/counting-principles

Counting & Quantity Visuals
mathisvisual.com/series/counting-and-quantity

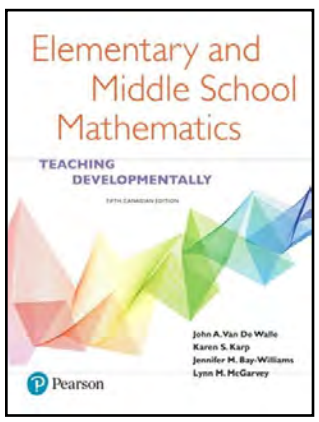
Count With Your Eyes: Subitizing Guide
tapintoteenminds.com/counting-with-your-eyes

How the 5 Counting Principles lay the foundation for flexible thinking in later grades
countingcollections.files.wordpress.com/2012/10/counting-principles-package.pdf

Mathematics in the Early Grades: Counting & Cardinality
interactivestem.org/wp-content/uploads/2015/09/Interactive-STEM-Brief-Counting-and-Cardinality-Sept-16-Final-File.pdf

The Principal Counting Principles
ncetm.org.uk/public/files/712850/The+principal+counting+principles.pdf

Books to Learn More About Counting, Quantity and Early Mathematics



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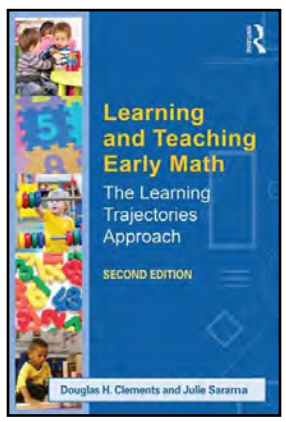

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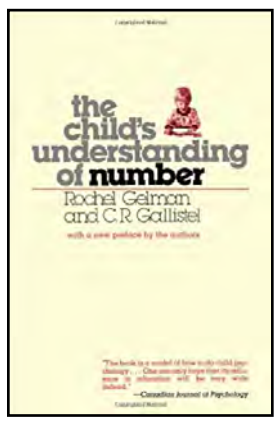

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Research

Bermejo, V., Morales, S. and Garcia de Osuna, J. (2004) Supporting children's development of cardinality understanding, Learning and Instruction, 14: 381–98.
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DCSF (Department for Children, Schools and Families) (2009), Children thinking mathematically: PSRN essential knowledge for Early Years practitioners. Nottingham: DCSF.
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Gelman, R. & Gallistel, C. (1978) The Child's Understanding of Number. Cambridge, MA. Harvard University Press.
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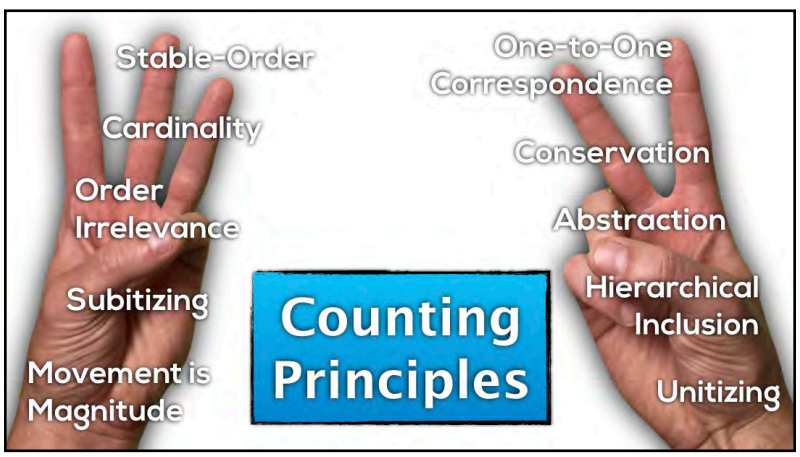
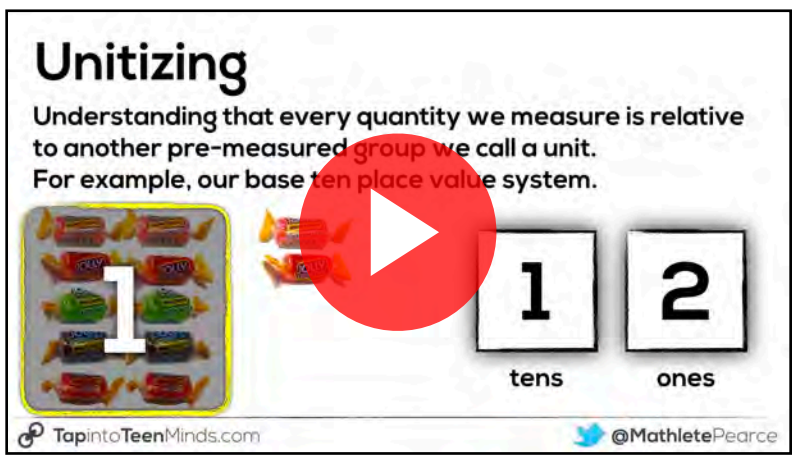
Clements, D. & Sarama, J. (2010) Learning Trajectories in Early Mathematics – Sequences of Acquisition and Teaching. Buffalo, NY
child-encyclopedia.com/sites/default/files/textes-experts/en/784/learning-trajectories-in-early-mathematics-sequences-of-acquisition-and-teaching.pdf

I hope you found this Principles of Counting and Quantity Cheat Sheet E-Book helpful in our journey to better understand the importance of early number and how we might help our children gain these important skills as they progress along their mathematical pathway.

Want to Visualize Counting Principles?

Watch the Counting & Quantity Animations





WATCH THE VIDEOS

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About Kyle Pearce

I'm Kyle Pearce and I am a former high school math teacher. I'm now the K-12 Mathematics Consultant with the Greater Essex County District School Board, where I uncover creative ways to spark curiosity and fuel sense making in mathematics. **Read more**.

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